PART – II: MATHEMATICS

SECTION - I **(Single Correct Choice Type)**

This is section contains **8 multiple questions**. Each question has four choices A), B), C) and D) out of which **ONLY ONE** is correct.

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20. Let f be a real – valued function defined on the interval (-1, 1) such that $e^{-x}f(x) = 2 + \int_0^x \sqrt{t^4+1}dt,$ for all $x \epsilon (-1,1)$ $and\ let\ f^{-1} be$ the inverse function of f. Then (f^{-1})(2) is equal to

A) 1 B) 1/3 C) 1/2 D) 1/e

Ans.: (B)

Sol. $f(x) = e^{-x}\left(2 + \int_0^x \sqrt{t^4+1}dt\right)$

Let g(x) = f^{-1}(x) $\Rightarrow g(f(x)) = x$

$\Rightarrow g'(f(x))f'(x) = 1$

$\Rightarrow g'^{(2)} = \frac{1}{f'(0)}$ $(\because f(0) = 2)$

Now f′(x) = $e^x\left(2 + \int_0^x \sqrt{t^4+1}dt\right) + e^x\sqrt{t^4+1}$ $(Applying\ Leibinitz\ Rule)$

$\Rightarrow f'(0) = 2 + 1 = 3$

$\Rightarrow g'(2) = \frac{1}{3}$

$\Rightarrow (f^{-1})'^{(2)} = \frac{1}{3}$

21. A single which can be given or red with probability 4/5 and 1/5 respectively, is received by station A and then transmitted to station B. The probability of each station receiving the single correctly is ¾. If the signal received at station B is green, then the probability that the original signal was green is

A) $\frac{3}{4}$ B) $\frac{6}{7}$ C) $\frac{20}{23}$ D) $\frac{9}{20}$

Ans.: (C)

Sol. Probability (P) = $\frac{P(GGG)+P(GRG)}{P(GGG)+P(GRG)+P(RGG)+P(RRG)}$

$\Rightarrow$ $P = \frac{\frac{4}{5}\times\frac{3}{4}\times\frac{3}{4}+\frac{4}{5}\times\frac{1}{4}\times\frac{1}{4}}{\frac{4}{5}\times\frac{3}{4}\times\frac{3}{4}+\frac{4}{5}\times\frac{1}{4}\times\frac{1}{4}+\frac{1}{5}\times\frac{1}{4}\times\frac{3}{4}+\frac{1}{5}\times\frac{3}{4}\times\frac{1}{4}}$

$\Rightarrow \qquad P = \frac{36+4}{36+4+3+3} = \frac{40}{46} = \frac{20}{23}$

22. if the distance of the point P(1, -2, 1) from the plane $x + 2y - 2z = \alpha, where\ \alpha > 0, is$ 5, then the foot of the perpendicular from P to the plane is

A) $\left(\frac{8}{3}, \frac{4}{3}, -\frac{7}{3}\right)$ B) $\left(\frac{4}{3}, -\frac{4}{3}, \frac{1}{3}\right)$ C) $\left(\frac{1}{3}, \frac{2}{3}, \frac{10}{3}\right)$ D) $\left(\frac{2}{3}, -\frac{1}{3}, \frac{5}{2}\right)$

Ans.: (A)

Sol. $D = \left|\frac{1-4-2-\alpha}{3}\right| = 5$

$\alpha + 5 = 15 \qquad (\therefore \quad \alpha > 0)$

$\Rightarrow \alpha = 10$

$\Rightarrow plane\ is\ x + 2y - 2z - 10 = 0$

For positive be (α,β,γ)

$\frac{\alpha-1}{1} = \frac{\beta+2}{2} = \frac{\gamma-1}{-2} = -\left(\frac{1-4-2-10}{9}\right) = \frac{5}{3} \quad \Rightarrow \alpha = \frac{8}{3}, \beta = \frac{4}{3}, \gamma = -\frac{7}{3}$

23. Let $S = (1, 2, 3, 4)$. The total number of unordered pairs of disjoint subsets of S is equal to

A) 25 B) 34 C) 42 D) 41.

Ans.: (D)

Sol. S = {1, 2, 3, 4}

Each element can be put in 3 ways either in subsets or we don't put in any subset.

So total number of unordered pairs = $\frac{3\times3\times3\times3-1}{2} + 1 = 41$. (Both subsets can be empty also)

24. For r = 0, 1, ..., 10. Let A_r , B_r and C_r denote, respectively, the coefficient of x^r in the expansions of (1 + *x)10,(1+x)20 and (1+x)30.* Then *r=110Ar(B10Br−C10Ar)*

A) $B_{10} - C_{10}$ B) $A_{10}(B_{10}^2 - C_{10}A_{10})$ C) 0 D) $C_{10} - B_{10}$

Ans.: (D)

Sol. $\sum_{r=1}^{10} A_r B_r - C_{10} \sum_{r=1}^{10} (A_r)^2 = B_{10}^{20}\,(C_{20}^{30} - 1) - C_{10}^{30}(C_{10}^{20} - 1) = C_{10}^{30} - C_{10}^{20} = C_{10} - B_{10}$

[By sum of series product of two binomial coefficients]

25. Two adjacent sides of a parallelogram ABCD are given by $\overrightarrow{AB} = 2\hat{i} + 10\hat{j} + 11\hat{k}$ and $\overrightarrow{AD} = -\hat{i} + 2\hat{j} + 2\hat{k}$. The side AD is routed by an acute angle with the side AB, then the cosine of the angle α is given by

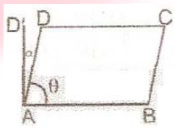
A) $\frac{8}{9}$ B) $\frac{\sqrt{17}}{9}$ C) $\frac{1}{9}$ D) $\frac{4\sqrt{5}}{9}$

Ans.: (B)

Sol. $\cos\theta = \frac{-2+20+22}{15\times 3} = \frac{8}{9}$ $[Using\ dot\ product]$

$\theta + \alpha = 90^0$

$\alpha = 90^0 - \theta$



$\cos\alpha = \sin\theta = \frac{\sqrt{17}}{9}$.

SECTION - II

(Integer Type)

This Section contains 5 questions. The answer to each question is a single – digit integer, reading from 0 to 9. The correct digit below question no. In the ORS is to be bubbled.

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26. Let k be a positive real number and let:

$$A = \begin{bmatrix} 2k-1 & 2\sqrt{k} & 2\sqrt{k} \\ 2\sqrt{k} & 1 & -2k \\ -2\sqrt{k} & 2k & -1 \end{bmatrix} \text{ and } B = \begin{bmatrix} 0 & 2k-1 & \sqrt{k} \\ 1-2k & 0 & 2\sqrt{k} \\ -\sqrt{k} & -2\sqrt{k} & 0 \end{bmatrix}$$

If det (adj A) + det (adj B) = 10^6, Then [k] is equal to

[Note: adj M denotes the Adjoint of square matrix M and [k] denotes the largest integer less than or equal to k].

Ans.: 4

Sol. $\det(A) = \begin{bmatrix} 2k-1 & 2\sqrt{k} & 2\sqrt{k} \\ 2\sqrt{k} & 1 & -2k \\ -2\sqrt{k} & 2k & -1 \end{bmatrix}$ $\quad C_2 \to C_2 - C_3$

$= \begin{bmatrix} 2k-1 & 0 & 2\sqrt{k} \\ 2\sqrt{k} & 1+2k & -2k \\ -2\sqrt{k} & 2k+1 & -1 \end{bmatrix}$ $\quad R_2 \to R_2 - R_3$

$= \begin{bmatrix} 2k-1 & 0 & 2\sqrt{k} \\ 4\sqrt{k} & 0 & 1-2k \\ 2\sqrt{k} & 2k+1 & -1 \end{bmatrix} = (2k+1)^3$

∴ B is a skew-symmetric matrix of odd order therefore det(B) =0

$Now \quad \det(adj\ A) + \det(adj\ B) = 10^6$

⇒ $\{(2k + 1)^3\}^2$ + det (adj B) $=10^6$

⇒ 2k + 1 = 10, as k > 0

⇒ k = 4.5

⇒ [k] = 4

27. Let f be a function defined on R (the set of all real numbers) such that $f(x) = 2010 (x - 2009)(x-2010)^2 (x-2011)^3 (x - 2012)^4$, for all $x \epsilon R$. If g is a function defined on R with values in the interval (0, ∞) such that f(x) = ln (g(x)), for all $x \epsilon R$.

Then the number of points in R at which g has a local maximum is

Ans.:1

Sol. $f'(x) = 2010 (x - 2009)(x-2010)^2 (x-2011)^3 (x - 2012)^4$

f(x) = ln (g(x))

$\Rightarrow g(x) = e^{f(x)}$

$\Rightarrow g'^{(x)} = e^{f(x)}..f'(x)$

Only point of maxima [Applying first derivative test]

28. Let $a_1, a_2, a_3, \dots, a_{11}$ be real numbers satisfying:

$a_1 = 15$, $27 - 2a_2 > 0$ and $a_k = 2a_{k-1} - a_{k-2}$ for k = 3, 4 ... 11.

If $\frac{a_1^2+a_2^2+\dots+a_{11}^2}{11} = 90.\ then\ the\ value\ of\ \frac{a_1+a_2+\dots+a_{11}}{11}\ is\ equal\ to$

Ans.: 0

Sol. $a_1 = 15$

$$\frac{a_k + a_{k-2}}{2} = a_{k-1} \; for \; k = 3, 4, \dots, 11$$

$\Rightarrow \; a_1, a_2, \dots\dots, a_{11} \; are \; in \; AP$

$a_1 = a = 15$

$\frac{a_1^2 + a_2^2 + \cdots + a_n^2}{11} = 90 \quad \Rightarrow \quad \frac{(15)^2 + (15+d)^2 + \cdots + (15+10d)^2}{11} = 90$

$\Rightarrow 9d^2 + 30d + 27 = 0 \quad \Rightarrow \; d = -3 \; or - \frac{9}{7}$

Since $27 - 2a_2 > 0 \quad \Rightarrow \; a_2 < \frac{27}{2} \quad \Rightarrow d = -3$

$$\frac{a_1 + a_2 + \cdots + a_{11}}{11} = \frac{11}{2} \frac{[30 + 10(-3)]}{11} = 0$$

29. Consider a triangle ABC and let a, b and c denote the length of the sides opposite to vertices A, B and C, respectively. Suppose a = 6, b = 10 and the area of the triangle is $15\sqrt{3}$. if $\angle ACB$ is obtuse and if r denotes the radius of the in circle of the triangle. Then r^2 is equal to

Ans.: 3

Sol. Area of triangle = $\frac{1}{2} \, ab \sin C = 15\sqrt{3}$

$\Rightarrow \frac{1}{2} .6.10 \sin C = 15\sqrt{3}$

$\Rightarrow \sin C = \frac{\sqrt{3}}{2}$

$\Rightarrow C = \frac{2\pi}{3} \qquad\qquad (C \; is \; obtuse \; angle)$

Now $\cos C = \frac{a^2 + b^2 - c^2}{2ab}$

$\Rightarrow -\frac{1}{2} = \frac{36 + 100 - c^2}{2.6.10} \quad \Rightarrow c = 14$

$\therefore r = \frac{\Delta}{s} = \frac{15\sqrt{3}}{\frac{6+10+14}{2}} = \sqrt{3}$

$\Rightarrow \; r^2 = 3$

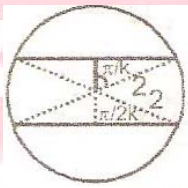
30. Two parallel chords of a circle of radius 2 are at a distance $\sqrt{3} + 1$ apart. If the chords subtend at the centre, angles of $\frac{\pi}{k} \; and \; \frac{2\pi}{k}$, where k > 0, then the value of [k] is

[Note: [k] denotes the largest integer less than or equal to k]

Ans.: 3

Sol. Since distance between parallel chords is greater than radius, therefore both chords lie on opposite side of centre.

$2\cos\frac{\pi}{2k}+2\cos\frac{\pi}{k}=\sqrt{3}+1$



Let $\frac{\pi}{2k}=\theta$

$\therefore 2\cos\theta+2\cos 2\theta=\sqrt{3}+1$

$\Rightarrow 2\cos\theta+2(2\cos^2\theta-1)=\sqrt{3}+1$

$\Rightarrow 4\cos^2\theta+2\cos\theta-\left(3+\sqrt{3}\right)=0$

$\therefore \quad \cos\theta=\frac{-2\pm\sqrt{4+16(3+\sqrt{3})}}{2(4)}=\frac{-2\pm2\sqrt{1+12+4\sqrt{3}}}{2(4)}=\frac{-1\pm\sqrt{(\sqrt{12}+1)^2}}{4}=\frac{-1\pm(2\sqrt{3}+1)}{4}$

$\Rightarrow \frac{\pi}{2k}=\frac{\pi}{6} \quad \Rightarrow k=3 \Rightarrow [k]=3$

Section-iii (paragraph Type)

This section contains 2 paragraphs. Based upon each of teh paragraphs 3 multiple choice questions have to be answered. Each of these question has four choices A),B),C) and D) out of which ONLY ONE is correct.

Paragraph for questions 31 to 33

Consider the polynomial

$f(x) = 1+2x+3x^2+4x^3$.

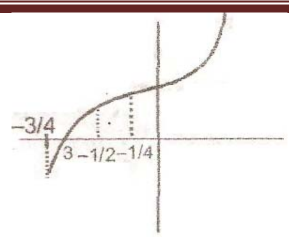
Let s be the sum of all distinct real roots of f(x) and let t=|s|.

31. The real number s lies in the interval

A) (-1/4 ,0) B) (-11, -3/4) C) (-3/4, -1/2) D) (0, ¼)

Ans.: (C)

Sol. $f(x)=1+2x+3x^2+4x^3$



$f'^{(x)} = 2 + 6x + 12x^2 > 0 \qquad [as\ a > 0, D < 0]$

f(x) is increasing function so it can almost one real root.

Using inter mediate value theorem

$f\left(-\frac{3}{4}\right).f\left(-\frac{1}{2}\right) < 0$

$\therefore (c)\ is\ correct.$

32. The area bounded by the curve y = f(x) and the lines x=0, y=0 and x = t, lies in the interval

A) (3/4 ,3) B) (21/64, 11/16) C) (9,10) D) (0, 21/64)

Ans.: (A)

Sol. By estimation of integration

$\int_0^{1/2} f(x)\,dx < \int_0^1 f(x)\,dx < \int_0^{3/4} f(x)\,dx$

$\Rightarrow \frac{15}{16} < \int_0^1 f(x)\,dx < \frac{525}{256}$

Hence option (A) is correct.

33. The function f'(x) is

A) increasing in (-t, -1/4) and decreasing in (-1/4,t) B) decreasing in (-t,-1/4) and increasing in (-1/4,t)

C) increasing in (-t, t) D) decreasing in (-t, t)

Ans.: (B)

Sol. $f'(x) = 2 + 6x + 12x^2$

$\Rightarrow f''(x) = 6 + 24x$

$\Rightarrow f''(x) = 6 + (4x + 1) > 0 \Rightarrow x > -\frac{1}{4}$

Paragraph for question 34 to 36

Tangents are drawn from the point P(3,4) to the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ touching the ellipse at points A and B.

34. The coordinates of A and B are

A) (3,0) and (0,2) B) $\left(-\frac{8}{5}, \frac{2\sqrt{161}}{15}\right)$ *and* $\left(-\frac{9}{5}, \frac{8}{5}\right)$

C) $\left(-\frac{8}{5}, \frac{2\sqrt{161}}{15}\right)$ *and* $\left(-\frac{9}{5}, \frac{8}{5}\right)$ D)(3,0) and (-9/5,8/5)

Ans.: (D)

Sol. Equation of chord of contact

$\frac{x}{3} + y = 1$

$x = 3(1 - y)$

Solving with ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$

$(1 - y)^2 + \frac{y^2}{4} = 1$

$4(y^2 + 1 - 2y) + y = 4$

$4y^2 - 8y = 0$

$y = 0$ & $\frac{8}{5}$

$\Rightarrow$ $x = 2$ & $3\left(1 - \frac{8}{5}\right) \Rightarrow x = 3, -\frac{9}{5}$

$\Rightarrow$ Points are (3, 0) and $\left(-\frac{9}{5}, \frac{8}{5}\right)$

35. The orthocentre of the triangle PAB is

A) (5,8/7) B) (7/5, 25/8) C) (11/5, 8/5) D) 8/25, 7/5)

Ans.: (C)

Sol. y-coordinate of the orthocentre must be $\frac{8}{5}$.

36. The Equation of the locus of the point whose distances from the point P and the line AB are equal, is

A) $9x^2 + y^2 - 6xy - 54x - 62y + 241 = 0$ B) $x^2 + 9y^2 + 6xy - 54x + 62y - 241 = 0$

C) $9x^2 + 9y^2 - 6xy - 54x - 62y - 241 = 0$ D) $x^2 + y^2 - 2xy + 27x + 31y - 120 = 0$

Ans.: (A)

Sol. $\sqrt{(x-3)^2 + (y-4)^2} = \frac{|x+3y-3|}{\sqrt{1+9}}$

$\Rightarrow 10\{(x^2 + 9 - 6x) + (y^2 + 16 - 8y)\} = (x + 3y - 3)^2$

$= x^2 + 9y^2 + 9 + 6xy - 6xy - 6x - 18y$

$\Rightarrow 9x^2 + y^2 - 6xy - 54x - 62y + 241 = 0$

SECTION –IV (Matrix Type)

This section contains 2 questions .each question has statements (A,B,C and D) given in column I and five statements (p, q, r, s and t) in column II. Any given statement in column I can have correct matching with one or more statement(s) given in column II. For examples, if for a given question, statements B matches with the statements given in q and r, then for that particular question, against statement, darken the bubbles corresponding t o q and r in the ORS .

37. match the statements in columns-I with those in column-II.

[NOTE: here z takes values in the complex plane and Im z and Re z denote, respectively, the imaginary part and the real part of z.]

Column I	Column II
A) The set of points z satisfying |z-i|z||=|z+i|z|| Is contained in or equal to **Ans.: q,r**	p) an ellipse with eccentricity 4/5
B) The set of points z satisfying |z+4|+|z-4|=10 Is containing in or equal to **Ans.: p**	q) the set of points z satisfying Im z = 0
C) If |w| = 2, then the set f points Z= w - $\frac{1}{w}$ is contained in or equal to **Ans.: p,s,t**	r)the set of points z satisfying |Im z| ≤ 1
D) If |w| =1, then the set f points Z= w + $\frac{1}{w}$ is contained in or equal to **Ans.: q,r,s,t**	s) the set of points z satisfying |Re z | ≤ 2
	t) the set od points z satisfying |z|≤3

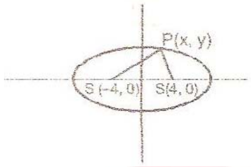
Sol. (A) $|(z - i|z|\ )| = |(z + i|z|\ )|$

$\Rightarrow\ |x + iy - i\sqrt{x^2+y^2}| = |x + iy + i\sqrt{x^2+y^2}|$

$\Rightarrow\ x^2 + (y - \sqrt{x^2+y^2})^2 = x^2 + (y + \sqrt{x^2+y^2})^2$

$\Rightarrow\ 4y\sqrt{x^2+y^2} = 0 \quad \Rightarrow y = 0 \Rightarrow Im\, z = 0$

(B) $|z+4| + |z-4| = 10$



Ellipse with $2a = 10\ \Rightarrow a = 5$

$ae = 4\ \Rightarrow e = \frac{4}{5}$

(C) Let $w = 2(\cos\theta + i\sin\theta)$

$z = 2(\cos\theta + i\sin\theta) - \frac{(\cos\theta + i\sin\theta)}{2}$

$= \frac{3\cos\theta + 5i\sin\theta}{2} \quad \Rightarrow \frac{3\cos\theta}{2}, y = \frac{5\sin\theta}{2}$

$= \frac{x^2}{9/4} + \frac{y^2}{25/4} = 1 \qquad e = \frac{4}{5}$

$|z| = \sqrt{\frac{9\cos^2\theta}{4} + \frac{25\sin^2\theta}{4}} = \sqrt{\frac{9+16\sin^2\theta}{4}} = \sqrt{\frac{9}{4} + 4\sin^2\theta} \le \frac{5}{2}$

$|Re\, z| = \left|\frac{3}{2}\cos\theta\right| \le \frac{3}{2}$

(D) $z = \cos\theta + i\sin\theta + \cos\theta - i\sin\theta = 2\cos\theta$

$\therefore |z| \le 2$

$\therefore Im(z) = 0$

$|Re\, z| \Rightarrow |2\cos\theta| \le 2$

$|z| \le 2$

38. Match the statements in column-I with the values in column-II.

Column I	Column II
A) A line from the origin meets the lines $\frac{x-2}{1} = \frac{y-1}{-2} = \frac{z+1}{1}$ & $\frac{x-\frac{8}{3}}{2} = \frac{y+3}{-1} = \frac{z-1}{1}$ at P and Q respectively. If	P) -4

length PQ = d, then d^2 is **Ans.: t**	
B)The Values of x satisfying $Tan^{-1}(x+3) – tan^{-1}(x-3) = sin^{-1}(3/5)$ are **Ans.:p,r**	q) 0
C) Non- zero vectors $\vec{a}, \vec{b}$ and $\vec{c}$ satisfy $\vec{a}.\vec{b} = 0$, $(\vec{b}-\vec{a}).(\vec{a}-\vec{b}) = 0$ and $2|\vec{b}+\vec{c}| = |\vec{b} - \vec{a}|$. If $\vec{a} = \mu\vec{b}+4\vec{c}$, then the possible values of μ are **Ans.: q,s**	r)4
D) Let f be the function on [-π, π] given by f(0) = 9 and f(x) = $\sin\left(\frac{9x}{2}\right) / \sin\left(\frac{x}{2}\right)$ *for* x≠ 0. The value of $\frac{2}{\pi}\int_{-\pi}^{\pi} f(x)dx$ *is* **Ans.: r**	s) 5
	t) 6

Sol. (A) Let the line through origin is $\frac{x}{\lambda} = \frac{y}{\mu} = \frac{z}{1}$

$\Rightarrow x = \lambda z, y = \mu z$ .... (1)

To the point of intersection of line (1) and line $\frac{x-2}{1} = \frac{y-1}{-2} = \frac{z+1}{1}$ .... (2)

We have $\frac{\lambda z-2}{1} = \frac{\mu z-1}{-2} = z + 1$

$\Rightarrow z = \frac{3}{\lambda-1} = \frac{-1}{\mu+2}$

$\Rightarrow z = \lambda + 3\mu + 5 = 0$ .... (3)

To find of intersection of line (1) and line $\frac{x-\frac{8}{3}}{2} = \frac{y+3}{-1} = \frac{z-1}{1}$ .... (4)

We have $\frac{\lambda z-\frac{8}{3}}{2} = \frac{\mu z+3}{-1} = \frac{z-1}{1}$

$\Rightarrow z = \frac{2}{3(\lambda-2)} = \frac{-2}{\mu+1}$

$\Rightarrow 3\lambda + \mu = 5$ .... (5)

Solving (3) and (5), $\lambda = \frac{5}{2}$ *and* $\mu = -\frac{5}{2}$

$\therefore z = 2, x = 5, y = -5 \; for \; point \; P$

$and \; z = \frac{4}{3}, x = \frac{10}{3}, y = -\frac{10}{3} \; for \; point \; Q$

$\therefore PQ^2 = \frac{4}{9} + \frac{25}{9} + \frac{25}{9} = 6$

(B) $\tan^{-1}(x+3) - \tan^{-1}(x-3) = \sin^{-1}(3/5)$

$\Rightarrow \tan^{-1}\left(\frac{x+3-x+3}{1+x^2-9}\right) = \tan^{-1}(3/4)$

$\Rightarrow \frac{6}{x^2-8} = \frac{3}{4} \quad \Rightarrow x^2 = 16$

$\therefore \quad x = \pm 4$

(C) Since$\vec{a}.\vec{b} = 0$

$\therefore \quad \vec{b} = \lambda_1 \hat{\imath}, \vec{a} = \lambda_2 \hat{\jmath}$

Now $2|\vec{b} + \vec{c}| = |\vec{b} - \vec{a}|$ & $\vec{a} = \mu\vec{b} + 4\vec{c}$

$\Rightarrow 2\left[\lambda_1 \hat{\imath} + \frac{\lambda_2 \hat{\jmath} - \lambda_1 \mu \vec{b}}{4}\right] = [\lambda_1 \hat{\imath} + \lambda_2 \hat{\jmath}]$

$|\lambda_1(4-\mu)\hat{\imath} + \lambda_2 \hat{\jmath}| = 2|\lambda_1 \hat{\imath} + \lambda_2 \hat{\jmath}|$

Squaring

$\lambda_1^2(4-\mu)^2 + \lambda_2^2 = 4\lambda_1^2 + 4\lambda_2^2$

$\Rightarrow 3\lambda_2^2 = (12 + \mu^2 - 8\mu)\lambda_1^2$ ………………(1)

Also $(\vec{b} - \vec{a}).(\vec{b} + \vec{c}) = 0$

$\Rightarrow (\lambda_1 \hat{\imath} - \lambda_2 \hat{\jmath}).\left(\lambda_1 \hat{\imath} + \frac{\lambda_2 \hat{\jmath} - \lambda_1 \mu \hat{\imath}}{4}\right) = 0$

$\Rightarrow \frac{\lambda_1^2(4-\mu)^2 + \lambda_2^2}{4} = 0$

$\Rightarrow \lambda_2^2 = \lambda_1^2(4-\mu)$ ……………. (2)

From (1) & (2)

$12 + \mu^2 - 8\mu = 12 - 3\mu$

$\Rightarrow \quad \mu^2 - 5\mu = 0 \Rightarrow \mu = 0, 5$

(D) $I = \frac{2}{\pi}\int_{-\pi}^{\pi} \frac{\sin\frac{9x}{2}}{\sin\frac{x}{2}} dx = \frac{4}{\pi}\int_0^{\pi} \frac{\sin\frac{9x}{2}\cos\frac{x}{2}}{\sin\frac{x}{2}\cos\frac{x}{2}} dx = \frac{8}{\pi}\int_0^{\pi} \frac{\sin\frac{9x}{2}\cos\frac{x}{2}}{\sin x} dx$

$I = \frac{4}{\pi}\int_0^{\pi}\frac{\sin 5x - \sin 4x}{\sin x}dx$ ………….. (i)

$(using \int_0^b f(x)dx = \int_0^b f(a+b-x)dx)$

$= \frac{4}{\pi}\int_0^{\pi}\frac{\sin 5x - \sin 4x}{\sin x}dx$ …………… (ii)

Add (i) & (ii)

$I = \frac{4}{\pi}\int_0^{\pi}\frac{\sin 5x}{\sin x}dx$

Consider

$I_k - I_{k-2} = \frac{4}{\pi}\int_0^{\pi}\frac{\sin kx - \sin(k-2)x}{\sin x} = \frac{4}{\pi}\int_0^{\pi}\frac{\sin(k-1)x \sin x}{\sin x}$

$I_k = I_{k-2}$

So $I_5 = I_3 \Rightarrow I_5 = I_1 = \frac{4}{\pi}\int_0^{\pi} dx = 4$

Alter

Let $I = \frac{4}{\pi}\int_{-\pi}^{\pi}\frac{\sin\frac{9x}{2}}{\sin\frac{x}{2}}dx$

$I = \frac{4}{\pi}\int_{-\pi}^{\pi}\frac{\sin\frac{9x}{2}}{\sin\frac{x}{2}}dx$ …………….. (1) ($\because f(x)\ is\ even\ function$)

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